QUESTION Sketch the following sets and determine which are regions.

(a) $|z-2+i| \leq 1$.

(b) $|2z+3| > 4$.

(c) $\mathrm{Im} z > 3$

(d) $|z-4| \geq |z|$

(e) $0 \leq \mathrm{Arg} z \leq \frac{\pi}{4}$, $(z \neq 0.)$

ANSWER

(a) $|z-2+1| \leq 1$ is a disk center $2-i$, radius 1. As it includes the boundary, it is not open and so not a region. (in fact it is a closed set.)

(b) $|2z+3| > 4 \Leftrightarrow |z+\frac{3}{2}| > 2$. This defines a region exterior to a disc centre $-\frac{3}{2}$, radius 2. It does not include the boundary so it is an open set. Also it is connected and hence a region.

(c) $\mathrm{Im} z > 3$ is a half-plane not including the line $\mathrm{Im} z = 3$ so it is open and connected and hence a region.

(d) This is the set of points closer to zero than to 4 (including the line $x = 2$.) Thus it is the half-plane on the side of the line $x = 2$ containing 0. As it includes the line $x = 2$ it is not open and thus not a region.

(e) This set contains the line $\arg z = \frac{\pi}{4}$ so it is not open and hence not a region. Actually, its complement contains 0 but no neighbourhood of 0 so it's complement is not open so the set is also not closed.